Conjugate Heat Transfer in Channels with Internal Longitudinal Fins

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A computational study examines the fully developed laminar flow and heat transfer characteristics of a parallel-plate channel with internal, longitudinal, rectangular fins. The fins are integrally attached to the channel top wall, whose outer surface is heated uniformly. The bottom wall is adiabatic. The study determines the effects of varying the fin height, the fin thickness, the fin pitch, and the wall-to-fluid thermal conductivity ratio on the pressure drop in the channel and the heat transfer from the exposed channel wall and fin surfaces. The results of the conjugate heat transfer study, which are applicable to the design of heat exchangers, include the overall heat transfer, the channel pressure drop, the heat transfer per unit pumping power, and the distributions of the local temperature and velocity.

Nomenclature

A_i = area of inner surface of top wall, including fin surface
A_o = area of outer surface of top wall
A_r = A_o/A_i, $= s/(s + 2h)$ when $h < d$, $= s/(s - t_f + 2h)$ when $h = d$
D_h = d_h/d
d = height of channel flow cross-section
d_h = hydraulic diameter of channel, $= 2(sd - ht_f)/(s + h)$ when $h < d$, $= 2(sd - ht_f)/(s - t_f + h)$ when $h = d$
$d_{h,pp}$ = hydraulic diameter of channel when there is no fin, $= 2d$
dp/dz = streamwise pressure gradient
f = friction factor, $= \tau_s/(\rho \bar{w}^2/2) = (-dp/dz)d_h/(2\rho \bar{w}^2)$
h = fin height
$\bar{h}$ = average heat transfer coefficient, $= q''_{in} A_r/\bar{T}_s - T_b)$
h_{pp} = heat transfer coefficient for flow in channel when there is no fin, $= Nu_{pp} k_f/d_{h,pp}$
K = ratio of thermal conductivity of finned wall to that of flowing fluid
k_f = thermal conductivity of flowing fluid
$\bar{Nu}$ = average Nusselt number, $= \bar{h} d_h/k_f$
Nu_{pp} = Nusselt number for flow in parallel-plate channel with one insulated wall and the other subjected to streamwise uniform heat flux and uniform wall temperature at any cross section, = 5.385
Nu_r = normalized Nusselt number, $= \bar{Nu}/Nu_{pp}$
P_r = normalized pressure drop, $= (-dp/dz)/(-dp/dz)_{pp}$
P_w = wetted perimeter
p = pressure
Q_p = dimensionless overall heat transfer per unit pumping power, $= Q_r/P_r^{1/3}$
Q_r = total heat transfer ratio, $= (\bar{h}A_i)/(h_{pp}A_o)$
q''_{in} = prescribed rate of heat transfer per unit area on outer surface of top wall
Re = Reynolds number based on hydraulic diameter, $= \rho \bar{w} d_h/\mu$
s = fin pitch
T, T_b = temperature and bulk temperature, respectively
T_i = arbitrary inlet fluid temperature
$\bar{T}_s$ = average surface temperature on inner surface of top wall (including fin surface)
t_f, t_w = fin and wall thickness, respectively
$w, \bar{w}$ = streamwise and average streamwise velocity, respectively
x, y, z = coordinates
ϕ = dimensionless temperature, $= (T - T_i)/(q''_{in}\, d/k_f)$
ϕ_b = dimensionless bulk temperature, $= (T_b - T_i)/(q''_{in}\, d/k_f)$
$\bar{\phi}_s$ = dimensionless average surface temperature, on inner surface of top wall (including fin surface), $= (\bar{T}_s - T_i)/(q''_{in}\, d/k_f)$
Ω = dimensionless streamwise velocity, $= w/[(-dp/dz)d^2/\mu]$
$\bar{\Omega}$ = dimensionless average streamwise velocity, $= \bar{w}/[(-dp/dz)d^2/\mu]$
μ = dynamic viscosity of fluid
ρ = fluid density
τ_s = wall shear stress

Subscripts

pp = parallel-plate channel with no fin
$H1$ = boundary condition, streamwise uniform heat flux and uniform wall temperature on heated channel walls at any cross section

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Introduction

IN many situations, fins enhance the rates of heat transfer between surfaces and adjoining fluids. In many prior analyses of fins, the convective heat transfer coefficients were specified in advance and were assumed to be uniform along the fin surfaces. In reality, however, the heat transfer coefficient varies along the fin surfaces and is affected by the actual fluid flow and thermal boundary conditions. Thus, the heat transfer characteristics of the fins can be determined accurately only by taking into account the simultaneous conduction of heat in the fins and convective heat transfer between the fins and the flowing fluid.

This research investigates the interaction between conduction and convection heat transfer in the thermally fully developed laminar flow of a fluid in a parallel-plate channel with longitudinal, rectangular fins. The fins are integrally attached to the inner surface of the top wall of the channel. The flow cross section of a typical finned channel is shown in Fig. 1. The outer surface of the channel top wall is heated uniformly and the bottom wall is adiabatic. The primary goals of this

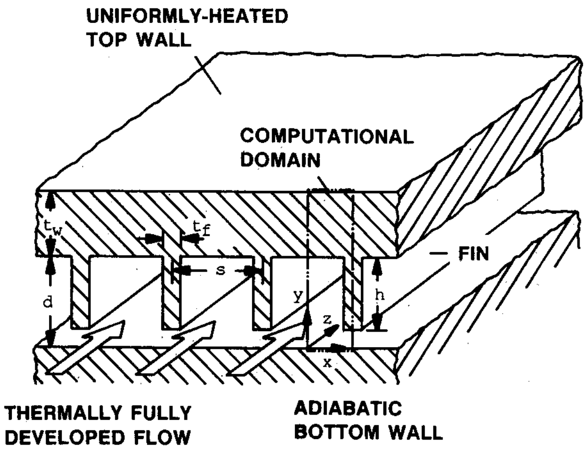

Fig. 1 Schematic cross-sectional view of finned channel.

study are to examine the effects of varying the flow channel geometry and the thermal conductivities of the wall and the flowing fluid on the heat transfer and flow characteristics of the channel, and to obtain overall heat transfer and pressure drop data for the design of heat exchangers.

In Sparrow et al.,[1] a numerical analysis studied the convective heat transfer from a shrouded array of thin (i.e., of negligible thickness) longitudinal fins. The flow was assumed to be laminar and thermally fully developed. One of two thermal boundary conditions was specified on the base surface: 1) uniform wall temperature and 2) streamwise uniform wall heat flux and uniform wall temperature at any cross section. Solutions were obtained for various interfin spacings, fin tip clearances, and fin conductance parameters.

The present investigation differs from Sparrow et al.[1] by including the study of the interaction between surface-fluid convective heat transfer and conduction in the fins as well as in the channel wall. The fins are of finite thicknesses and heat conduction in the fins is considered to be two-dimensional. Results are obtained for wide ranges of the wall-to-fluid thermal conductivity ratio and geometric parameters—for thin fins, thick fins, short fins, as well as tall fins, with small to large fin pitches.

Analysis

Because of geometric symmetry, the numerical calculation is performed for a typical module (Fig. 1). The calculation domain is bounded by the outer surface of the uniformly-heated top wall, the inner surface of the insulated bottom wall, and two symmetry planes, through the center plane of a fin and midway between two adjacent fins, respectively. Therefore, the calculation domain measures $(s/2)$ by $(d + t_w)$. Figure 1 displays the coordinates used in the analysis.

It is assumed that the thermophysical properties of the flowing fluid and those of the finned channel top wall (the channel top wall and the fins are of the same material) are constant, the flow is thermally fully developed and laminar, and wall conduction in the flow direction is negligible. The streamwise momentum equation is solved first. The resulting velocity field is then used to solve the energy equation. In the analysis, the following dimensionless parameters are employed:

$$X = x/d; \quad Y = y/d; \quad S = s/d;$$
$$H = h/d; \quad F = t_f/d; \quad W = t_w/d \tag{1}$$

The thermal conductivity ratio K is defined as the ratio of the thermal conductivity of the wall (also the fins) to that of flowing fluid.

For a fully developed laminar flow through the finned channel, the dimensionless streamwise momentum and energy conservation equations are

$$\left(\frac{\partial^2\Omega}{\partial X^2}\right) + \left(\frac{\partial^2\Omega}{\partial Y^2}\right) + 1 = 0 \tag{2}$$

$$\left(\frac{\partial^2\phi}{\partial X^2}\right) + \left(\frac{\partial^2\phi}{\partial Y^2}\right) - \left[\frac{S}{(S-HF)}\right]\left(\frac{\Omega}{\bar{\Omega}}\right) = 0 \tag{3}$$

where $\Omega = w/[(-dp/dz)d^2/\mu]$ and $\phi = (T - T_i)/(q''_{\text{in}} d/k_f)$.

The boundary conditions for Eq. (2) are $\Omega = 0$ on the solid walls and $(\partial\Omega/\partial X) = 0$ on the symmetry boundaries. The boundary conditions for Eq. (3) are $(\partial\phi/\partial X) = 0$ on the symmetry planes, $(\partial\phi/\partial Y) = 0$ on the bottom wall, and uniform heat flux on the outside surface of the top wall.

The governing equations are solved numerically by a control-volume-based finite difference scheme, which is described in Patankar.[2] In the solution of the momentum equation, the dynamic viscosity in the solid wall and fin regions is assumed to have a very large value, such that the velocity is equal to zero in the regions (see Ref. 3).

In each of the cases studied, computation is performed with a nonuniform grid of 42×52 nodal points in the X and Y directions, respectively. Many nodal points are used in various regions in the calculation domain where the velocity and temperature gradients are expected to be large. In general, a large number of nodal points are concentrated in the fluid region near the channel wall and fin surfaces, with a coarse grid of nodal points in the solid wall region, where the velocity is zero and the temperature gradient is small. For instance, in all cases with $W = 1$, there are 11 and 41 points (10 and 40 control volumes) along the Y axis in the solid region and in the fluid region, respectively.

For each parametric condition, the solutions of the momentum and energy equations consist of the detailed distributions of the dimensionless velocity, $(\Omega/\bar{\Omega})$, in the flow cross-section and the dimensionless temperature, $(\phi - \phi_b)$, in the finned channel wall and in the flow. From the velocity and temperature distributions, the friction factor-Reynolds number product, fRe, and the overall Nusselt number, $\bar{Nu}$, are obtained, as follows:

$$\begin{aligned} fRe &\equiv [\tau_s/(\rho\bar{w}^2/2)](\rho\bar{w}d_h/\mu) \\ &= [(-dp/dz)d_h/(2\rho\bar{w}^2)](\rho\bar{w}d_h/\mu) \\ &= (dp/dz)d_h^2/(2\mu\bar{w}) \\ &= D_h^2/(2\bar{\Omega}) \end{aligned} \tag{4}$$

$$\begin{aligned} \bar{Nu} &\equiv \bar{h}d_h/k_f \\ &= [q''_{\text{in}}A_r/(\bar{T}_s - T_b)]d_h/k_f \\ &= D_hA_r(\bar{\phi}_s - \phi_b) \end{aligned} \tag{5}$$

In order to provide data for design optimization, the overall heat transfer and pressure drop are normalized with their corresponding parallel-plate channel (with no fin) values for the same mass flow rate. The normalized pressure drop is defined as

$$\begin{aligned} P_r &\equiv (-dp/dz)/(-dp/dz)_{pp} \\ &= [(fRe)/(fRe)_{pp}](d_{h,pp}/d_h)^2(\bar{w}/\bar{w}_{pp}) \end{aligned} \tag{6}$$

Since $(fRe)_{pp} = 24$ and $d_{h,pp} = 2d$,

$$P_r = [(fRe)/(6D_h^2)][S/(S-HF)] \tag{7}$$

The total heat transfer ratio is defined as

$$Q_r \equiv (\bar{h}A_i)/(h_{pp}A_o)$$
$$= \{[q''_{in}A_r/(\bar{T}_s - T_b)]/[Nu_{pp}k_f/d_{h,pp}]\}/A_r \quad (8)$$

Since $Nu_{pp} = 5.385$ and $d_{h,pp} = 2d$,

$$Q_r = [(0.3714)/(\bar{\phi}_s - \phi_b)] \quad (9)$$

Finally, a dimensionless overall heat transfer per unit pumping power is defined as

$$Q_p \equiv Q_r/P_r^{1/3}$$
$$= [(\bar{h}A_i)/(h_{pp}A_o)]/[(-dp/dz)/(-dp/dz)_{pp}]^{1/3} \quad (10)$$

Equation (10) can be rewritten as

$$Q_p = \left[\frac{(\bar{h}/h_{pp})}{(f/f_{pp})^{1/3}}\right]\left\{\left(\frac{A_i}{A_o}\right)\left[\frac{(S-HF)}{S}\right]\Big/\left(\frac{P_w}{P_{w,pp}}\right)^{1/3}\right\} \quad (11)$$

where the last three terms are the ratios of the heat transfer surface area, the flow cross-sectional area, and the wetted perimeter for the finned channel to the corresponding geometric parameters for a plain parallel-plate channel with no fin, respectively.

Results and Discussion

The effects of varying the fin height, fin thickness, fin pitch and the wall-to-fluid thermal conductivity ratio on the overall heat transfer and pressure drop in the channel are discussed. Sample results of the distributions of the local temperature and velocity are presented and discussed briefly. The channel wall thickness effect is not presented, since results show that varying the channel wall thickness has little effect on the overall heat transfer. Detailed discussion of all of the results appears in a thesis by Ong.[4]

All of the computations were performed on a VAX 8650 computer. In each run, the iterative procedure was stopped when the values of fRe and the fluid bulk temperature, and the values of the velocity and temperature at two randomly-selected locations remained constant to six significant figures in at least 10 consecutive iteration steps. The computation procedure was validated further by obtaining the values of fRe and Nu_{H1} for thermally fully developed laminar flows through rectangular channels of various aspect ratios (by setting $H = 1$, varying F and S, and letting K be very large) with one adiabatic wall and the other three walls subjected to streamwise uniform heat flux and uniform wall temperature at any cross section. Table 1 shows the comparison between some of the values of fRe and Nu_{H1} and those published in Shah and London.[5]

In applying the numerical results of this study to the design of heat exchangers, a design engineer is cautioned that 1) the fins are assumed to be integrally attached to the inner surface of the top channel wall (contact resistance is neglected); 2) all properties are constant; 3) the flow is thermally fully developed and laminar; and 4) streamwise conduction is neglected. Depending on the manufacturing process, the fins may be in poor contact with the channel wall. As a result, the heat flow to the fins may be reduced considerably. Small variations of the properties of the finned channel wall and the fluid and wall conduction in the streamwise direction should not change the general trends of the results significantly.

Table 2 Fin height effect

H	P_r	Q_r	Q_p
$F = 0.05$, $S = 0.25$, $W = 0.25$, $K = 500$			
0.0	1.0	1.0	1.0
0.25	2.019	1.334	1.055
0.50	5.817	2.279	1.798
0.75	21.077	11.032	3.994
1.0	35.589	56.989	17.325

$F = 1.0$, $S = 2.0$, $W = 1.0$

F	P_r	$K = 25$		$K = 500$	
		Q_r	Q_p	Q_r	Q_p
0.0	1.0	1.0	1.0	1.0	1.0
0.25	1.662	1.257	1.061	1.275	1.076
0.50	2.997	1.759	1.220	1.818	1.261
0.625	3.801	1.931	1.237	1.991	1.276
0.75	4.416	1.954	1.191	1.996	1.217
0.875	4.691	1.953	1.167	1.977	1.181
1.0	4.731	1.936	1.153	1.988	1.184

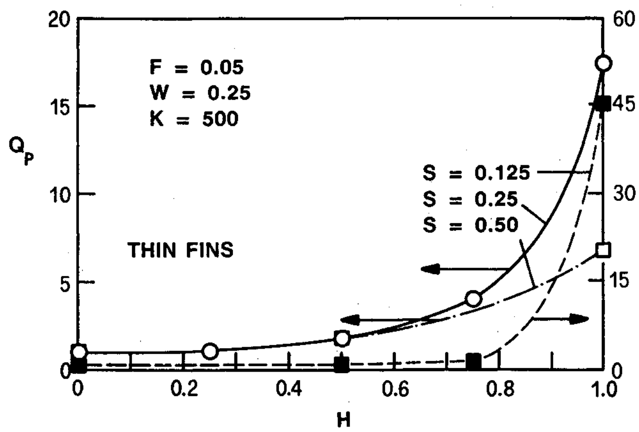

Fig. 2 Fin height effect, thin fins.

Effect of Fin Height

When the fins are shorter than the height of the channel, the clearance space beyond the fin tips is an alternative fluid flow path to that between adjacent fins. Since, by nature, the flow favors a path of least resistance, the flow velocity may be very

Table 1 Comparison of present results with data from Shah and London[5]

Fin height, H	Fin thickness, F	Fin spacing, S	Aspect ratio, $(S\text{-}F)/H$	Present values		Shah and London[5]	
				fRe	Nu_{H1}	fRe	Nu_{H1}
—	—	—	∞	23.970	5.385	24.00	5.385
1.0	0.25	2.0	1.75	15.095	3.166	15.142[a]	3.167[a]
1.0	1.0	2.0	1.0	14.193	3.570	14.227	3.556
1.0	0.5	1.0	0.5	15.506	4.523	15.548	4.505
1.0	0.05	0.50	0.45	15.874	4.728	15.958[a]	4.695[a]
1.0	0.75	1.0	0.25	18.101	5.757	18.233	5.733[a]
1.0	0.05	0.25	0.2	18.981	6.150	19.071	6.072

[a]From interpolation of available data.

high in the interfin spaces or in the clearance space, depending on the fin height, fin pitch, and fin thickness. The distribution of the flow between the interfin spaces and the clearance space affects significantly the overall pressure drop and the heat transfer from the channel wall and fin surfaces.

The overall heat transfer and pressure drop results are presented for 1) a thin-fin geometry with $F = 0.05$, $S = 0.25$, $W = 0.25$, and $K = 500$, and 2) a thick-fin geometry with $F = 1.0$, $S = 2.0$, $W = 1.0$, and $K = 25$ and 500. In Figs. 2 and 3, the overall heat transfer per unit pumping power Q_p is plotted vs H for the thin-fin and thick-fin cases, respectively. Table 2 gives the values of P_r, Q_r, and Q_p for various values of H in the two cases.

In Fig. 2, for a channel with closely spaced thin fins, increasing the fin height increases the heat transfer per unit pumping power. With increasing H, the value of Q_p increases monotonically. When $H = 1.0$, the heat transfer per unit pumping power of the finned channel is about 17 times that of a plain parallel-plate channel with no fin. A close examination of Table 2 reveals that, with H increasing from 0.0–0.75, P_r increases more rapidly than Q_r. There is a large jump in the value of Q_r, however, when H is increased from 0.75–1.0.

The numerical results also show that, when the fin pitch is varied, the monotonically increasing trend of Q_p with increasing H remains unchanged. When S is halved ($S = 0.125$, keeping other parameters the same), $Q_p = 1.033$, 1.359, and 45.159, for $H = 0.5$, 0.75, and 1.0, respectively. The value of Q_p increases substantially when H is increased from 0.75–1.0. When S is doubled ($S = 0.5$, other parameters unchanged), $Q_p = 1.756$ and 6.753 for $H = 0.5$ and 1.0, respectively. In the case of a large fin pitch ($S = 2.0$, other parameters unchanged), $Q_p = 1.234$ and 1.518 for $H = 0.5$ and 1.0.

In the case of a channel with thick fins, Fig. 3 shows that the value of Q_p increases with increasing fin height, reaches a maximum when $H \simeq 0.6$, then decreases and levels off gradually. The curve for $K = 500$ has the same shape as that for $K = 25$ and is always higher than that for $K = 25$. In these thick-fin cases, fins with $H \simeq 0.6$ provide the optimum heat transfer for the same pumping power. Table 2 shows that both P_r and Q_r increase with increasing H for $H < 0.625$. For the values of H larger than 0.625, Q_r remains almost constant whereas P_r increases gradually for $0.625 < H < 0.875$.

When S is doubled ($S = 4.0$, $K = 500$, other parameters unchanged), $Q_p = 0.915$, 0.849, and 0.843 for $H = 0.5$, 0.75, and 1.0, respectively. The value of Q_p decreases with increasing H. Since $Q_p < 1.0$ over the entire range of $0 < H \leq 1.0$, there is no advantage in installing these fins on the channel top wall.

The detailed local velocity and temperature distributions for two of the thick-fin cases considered here are presented in Figs. 4a through 4d. When the fins are short, the fin tip surface is washed by cool and fast-moving fluid. As a result, a relatively large percentage of the total heat input is transferred to the fluid from the fin tip surface. When H is large (but less than 1.0), the velocity of the fluid flowing over the fin tip surface is small and the temperature of the fluid near the fin tip surface is high. Consequently, the heat transfer from the fin tip surface is relatively small.

In the case of a channel with thin fins that are shorter than the channel height and are spaced closely, the fluid velocity is generally low and the fluid temperature is high in the interfin spaces, because of the relatively large flow resistance in the interfin spaces. The reverse is true in the clearance space.

In both the thick-fin and thin-fin cases, with $H < 1.0$, the surface heat flux is the highest and the surface temperature is the lowest at the corner at the fin tip.

Effect of Fin Thickness

For a channel with fins that have the same height as the channel ($H = 1.0$), increasing the fin thickness while keeping the fin pitch constant decreases the heat transfer per unit pumping power. Figure 5 shows that, for a finned channel

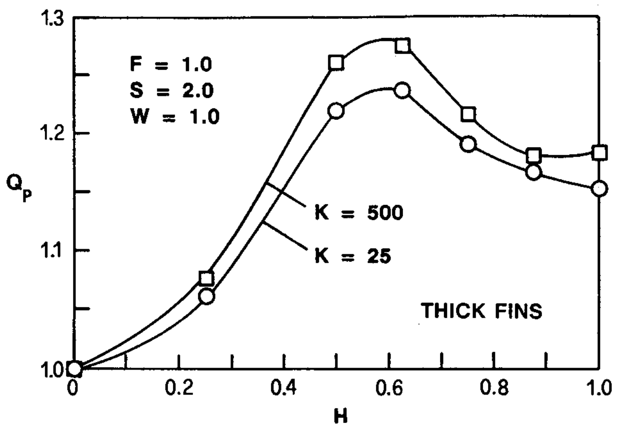

Fig. 3 Fin height effect, thick fins.

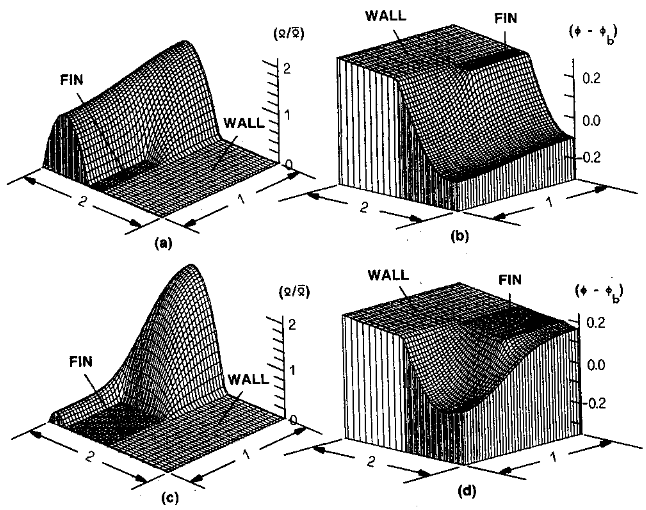

Fig. 4 Typical local velocity and temperature distributions, $F = 1.0$, $S = 2.0$, $W = 1.0$, $K = 25$: a) and b) $H = 0.25$; and c) and d) $H = 0.75$.

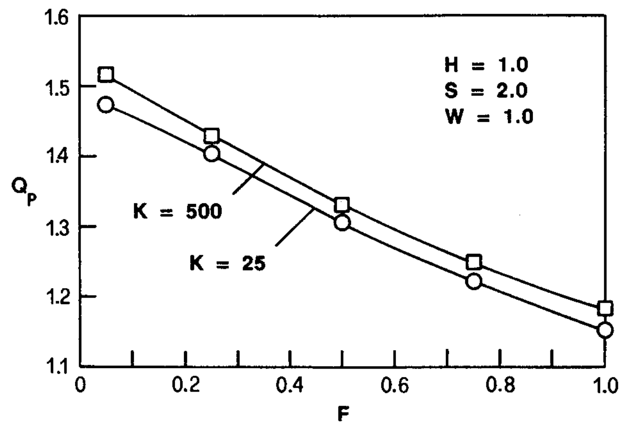

Fig. 5 Fin thickness effect.

with $H = 1.0$, $S = 2.0$, $W = 1.0$, and $K = 25$ and 500, the value of Q_p decreases monotonically with increasing F. Increasing the fin thickness from 0.05–1.0 increases Q_r slightly from about 1.7–2.0 (see Table 3). The value of P_r for $F = 1.0$, however, is more than three times that for $F = 0.05$.

In a channel with short fins ($H < 1.0$), the heat transfer per unit pumping power may increase with increasing fin thickness. For a channel with $H = 0.5$, $S = 2.0$, $W = 1.0$, and $K = 500$, increasing F from 0.05–1.0 increases Q_p from 1.234–1.261.

For a given interfin spacing, $(S\text{-}F)$, the effect of varying the fin thickness on the heat transfer characteristics of a channel that has fins with $H = 1.0$ is examined by comparing the numerical results in the following two cases: 1) a thin-fin array with $F = 0.05$, $H = 1.0$, $S = 1.05$, $W = 1.0$, and $K = 500$, and 2) a thick-fin array with $F = 1.0$, $H = 1.0$, $S = 2.0$, $W = 1.0$, and $K = 500$. The values of Q_p in the two cases are 2.898 and 1.184, respectively. The disadvantage of increasing the fin thickness for a given interfin spacing is reaffirmed by comparing the results in the following two cases: 1) a thin-fin array with $F = 0.05$, $H = 1.0$, $S = 0.55$, $W = 1.0$, and $K = 500$, and 2) a thick-fin array with $F = 0.5$, $H = 1.0$, $S = 1.0$, $W = 1.0$, and $K = 500$. The values of Q_p in the two cases are 6.391 and 2.782, respectively.

Effect of Fin Pitch

For a channel with an array of thin fins with $F=0.05$, $H=1.0$, $W=0.25$, and $K=500$, the values of Q_r, P_r, and Q_p were obtained for $0.125 \le S \le 4.0$. These values are tabulated in Table 4. When S is large, both the overall pressure drop and the overall heat transfer in the finned channel are only slightly higher than those for a plain parallel-plate channel with no fin. As the fin pitch is reduced, both P_r and Q_r increase rapidly. When $S = 0.5$, the pressure drop in the finned channel is about 8 times that for a plain parallel-plate channel; the overall heat transfer in the finned channel is about 13 times that for a plain channel with no fin. When $S = 0.125$, $P_r = 309$ and $Q_r = 305$.

Figure 6 shows the rapid increase in the overall heat transfer per unit pumping power as the fin pitch is decreased. When S is reduced from 2.0–1.0, the value of Q_p goes up from 1.518–2.895, an increase of about 90%.

For relatively thick fins with $H = 1.0$, $F = 0.25$, 0.5, and 0.75, the values of P_r, Q_r, and Q_p for $S = 1.0$ and 2.0 are given in Table 5. When S is reduced from 2.0–1.0, the increase in the heat transfer per unit pumping power is between 83 and 128%.

For thick fins with $F=1.0$, $W=1.0$, $K=500$, and $H=0.5$, 0.75, and 1.0, reducing the fin pitch from 4.0–2.0 increases Q_p from 0.915–1.261, from 0.849–1.217, and from 0.844–1.184, respectively. In each case, the increase in Q_p is about 40%.

When the height of the thin-fin array ($F = 0.05$, $W = 0.25$, and $K = 500$) is halved ($H = 0.5$), the effect of varying the fin pitch on the heat transfer per unit pumping power is changed drastically. Figure 6 shows that Q_p attains a maximum at $S \simeq 0.5$, and reducing S from 0.5–0.125 decreases Q_p from 1.756–1.033. Reducing the fin pitch when the fins are closely-spaced not only increases the pressure drop but also reduces the overall heat transfer (see Table 4).

Effect of Thermal Conductivity Ratio

Varying the thermal conductivity ratio affects the heat transfer in a channel with thin fins more than that in a channel with thick fins. Figure 7 shows that, for a thin-fin array of $F=0.05$, $H=1.0$, $S=0.25$, and $W=0.25$, increasing the value of K from 25–500 almost doubles the normalized heat transfer per unit pumping power, from 9.090–17.325. For $K>8000$, $Q_p \simeq 18.300$. For the same thin-fin array, when the fin pitch is doubled, the values of Q_p for $K=25$, 500, and ∞ are 4.309, 6.753, and 7.041, respectively.

In the case of a channel with thick fins ($F = 1.0$, $H = 1.0$, $S = 2.0$, and $W = 1.0$), the value of Q_p is almost independent of K. When K increases from 25–500, the value of Q_p increases from 1.153–1.184. For $K>500$, the value of Q_p remains constant.

Figures 8a and 8b display the detailed local temperature distributions for a thin-fin channel ($F = 0.05$, $H = 1.0$, $S = 0.25$, and $W = 0.25$) with $K = 25$, and 8000, respectively. In the figures, the dimensions along the X axis are highly exaggerated. When the value of K is small, the fin temperature and the fluid temperature decrease significantly with increasing distance from the top wall. When the value of K is large, the fin temperature drops only slightly along the fin length, and the fluid temperature is low in the middle of the flow cross section. For $K = 25$, about 39 and 61% of the total heat transfer is transferred from the exposed channel wall and from the fin, respectively. Increasing the value of K from 25–8000 increases the fin heat transfer by 48%, with a corresponding drop in the channel wall heat transfer. For $K>500$, the percentages of the total heat input that are transferred from the

Table 3 Fin thickness effect

H	P_r	$K = 25$ Q_r	$K = 25$ Q_p	$K = 500$ Q_r	$K = 500$ Q_p
$H = 1.0$, $S = 2.0$, $W = 1.0$					
0.05	1.510	1.691	1.474	1.740	1.517
0.25	1.775	1.701	1.405	1.732	1.431
0.50	2.266	1.717	1.307	1.750	1.332
0.75	3.099	1.782	1.222	1.822	1.250
1.00	4.731	1.936	1.153	1.988	1.184

Table 4 Fin pitch effect, thin fins

S	$H = 1.0$ P_r	$H = 1.0$ Q_r	$H = 1.0$ Q_p	$H = 0.5$ P_r	$H = 0.5$ Q_r	$H = 0.5$ Q_p
$F = 0.05$, $W = 0.25$, $K = 500$						
0.125	308.621	305.175	45.159	7.372	2.010	1.033
0.25	35.589	56.989	17.325	5.817	2.279	1.267
0.5	7.630	13.294	6.753	3.476	2.660	1.756
1.0	2.627	3.994	2.895	1.931	2.012	1.616
1.5	—	—	—	1.516	1.572	1.369
2.0	1.510	1.741	1.518	1.347	1.362	1.234
4.0	1.203	1.193	1.122	—	—	—

Table 5 Fin pitch effect, thick fins

F	S	P_r	$K = 25$ Q_r	$K = 25$ Q_p	$K = 500$ Q_r	$K = 500$ Q_p
$H = 1.0$, $W = 1.0$						
0.25	1.0	4.371	4.197	2.567	4.649	2.843
	2.0	1.775	1.701	1.405	1.732	1.430
0.5	1.0	11.630	5.796	2.558	6.303	2.782
	2.0	2.266	1.717	1.307	1.750	1.332
0.75	1.0	75.421	11.042	2.613	12.031	2.848
	2.0	3.099	1.782	1.222	1.822	1.250

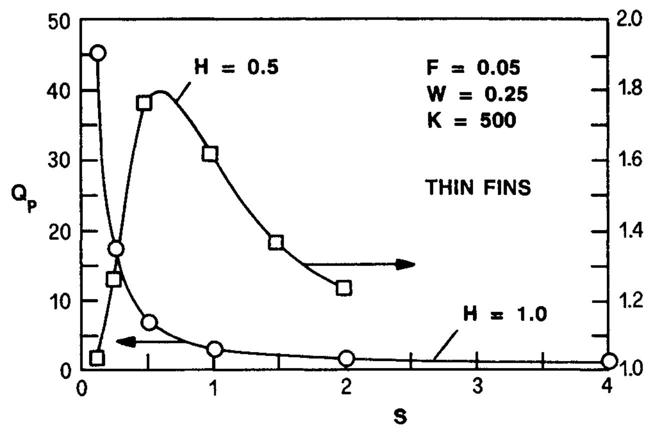

Fig. 6 Fin pitch effect.

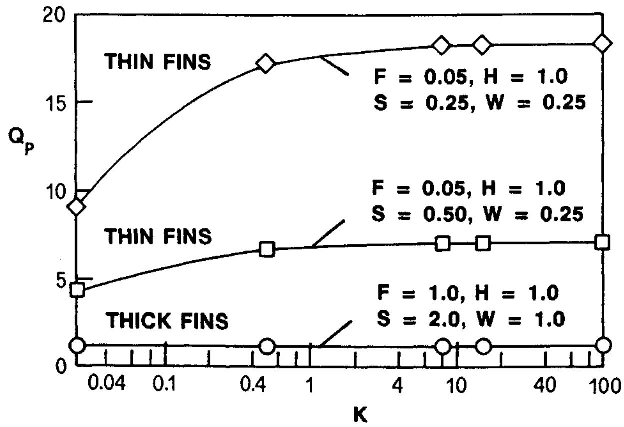

Fig. 7 Thermal conductivity ratio effect.

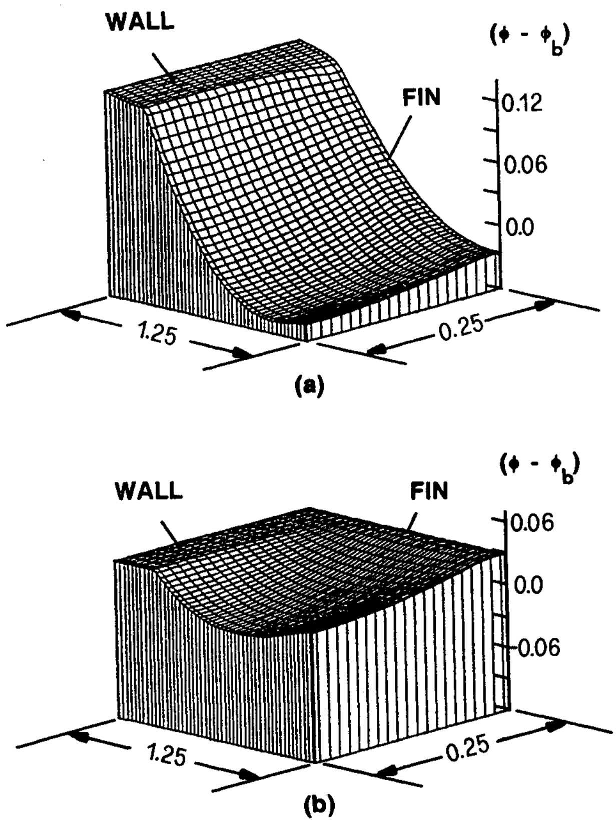

Fig. 8 Effect of K on local temperature distribution, $F = 0.05$, $H = 1.0$, $S = 0.5$, $W = 0.25$, a) $K = 25$; and b) $K = 8000$.

exposed channel surface and the fin surface are almost independent of the value of K.

Concluding Remarks

The thermally fully developed laminar flow and heat transfer characteristics of a parallel-plate channel with an adiabatic wall and a uniformly-heated wall with internal, longitudinal, rectangular fins have been studied systemically. Based on the results of the numerical study, the following conclusions can be drawn:

1) For a channel with closely-spaced thin fins, increasing the fin height increases the heat transfer per unit pumping power. In the case of a channel with thick fins, the heat transfer per unit pumping power may decrease with increasing fin height.

2) For a channel with fins that have the same height as the channel, increasing the fin thickness while keeping the fin pitch constant decreases the heat transfer per unit pumping power. With shorter fins, however, the heat transfer per unit pumping power may increase with increasing fin thickness.

3) The overall heat transfer per unit pumping power generally increases as the fin pitch is decreased. When the fins are short and closely-spaced, however, reducing the fin pitch not only increases the pressure drop but also reduces the overall heat transfer.

4) Varying the wall-to-fluid thermal conductivity ratio affects the heat transfer in a channel with thin fins more than that in a channel with thick fins. When the thermal conductivity ratio is larger than about 500, the heat transfer per unit pumping power does not increase significantly when the thermal conductivity ratio is increased.

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